Particle Swarm Optimization with Constriction Factor and Inertia Weight Approach Based Synthesis of Concentric Circular Antenna Array with Non-isotropic Elements

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***Abstract*—In this paper, an evolutionary optimization technique, Particle Swarm Optimization with Constriction Factor and Inertia Weight Approach (PSOCFIWA) is adopted for the complex synthesis of three-ring Concentric Circular Antenna Arrays (CCAA) with non-isotropic elements and without and with central element feeding. It is shown that by selection of a fitness function which controls more than one parameter of the array pattern, and also by proper setting of weight factors in fitness function, one can achieve very good results. For each optimal design, optimal current excitation weights and optimal radii are determined having the objective of maximum Sidelobe Level (SLL) reduction. The extensive computational results show that the CCAA designs having central element feeding with non-isotropic elements yield much more reduction in SLL as compared to the same not having central element feeding. Moreover, the particular CCAA containing 4, 6 and 8 number of elements in three successive rings along with central element feeding yields grand minimum SLL (-46.4 dB). Standard Particle Swarm Optimization (PSO) is adopted to compare the results of the PSOCFIWA algorithm.**

***Keywords*— Concentric Circular Antenna Array; Non-isotropic Elements; Particle swarm optimization; Non-uniform Excitation; Sidelobe Level; First Null Beamwidth**

I. Introduction

A Concentric Circular Antenna Array (CCAA) is an array which contains many concentric circular rings of different radii and a number of antenna elements located on the circumference of each ring [1, 2]. CCAA has received considerable interest due to its symmetricity and compactness in structure. Since a concentric circular array does not have edge elements, directional patterns synthesized with a concentric circular array can be electronically rotated in the plane of the array without a significant change of the beam shape. CCAA offers great flexibility in array pattern synthesis and design both in narrowband and broadband applications. These salient features of CCAA have made it indispensable in mobile and communication applications. These antenna arrays have improved the performance of mobile and wireless communication systems through efficient spectrum utilization, enhancing channel capacity, extending coverage area and tailoring beam shape etc. However, antenna array design can not be done on an arbitrary basis as this may lead to pollution of the electromagnetic environment and more importantly, wastage of precious power. The later may prove fatal for power-limited battery-driven wireless devices. This has encouraged a lot of research work [3-11] in the field of optimisation of antenna structures, all having a common objective of bridging the gap between desired radiation patterns with what is practically achievable.

The goal of this paper is to optimize current excitation weights and radii in order to minimize the SLL, hence working towards the improvement of antenna array pattern. In this approach the structural geometry of the antenna array is manipulated while preserving the gain of the main beam. This paper presents the design of concentric circular antenna array with non-isotropic elements. CCAA designs having non-isotropic elements (i) with and (ii) without central element feeding [12-13] have been considered in this paper. Many synthesis methods are concerned with suppressing the SLL, as the shape of the desired pattern can vary widely depending on the application. In this paper an optimal CCAA is designed with the help of Standard Particle Swarm Optimization (PSO) [14] and Particle Swarm Optimization with Constriction Factor and Inertia Weight Approach (PSOCFIWA) [15-16] techniques. The array factors due to optimal radii and non-uniform excitations of non-isotropic elements in various CCAA designs are examined to find the best possible design set.

II. Design Equation

Fig. 1 is an illustration of the general configuration of CCAA having M concentric circular rings, where the m^{th} ($m = 1, 2, \ldots, M$) ring has a radius r_m and the corresponding number of elements is N_m. If all the elements (in all the rings) are assumed to be isotropic sources, the radiation pattern of this array can be written in terms of its array factor only. But the elements considered in this paper are non-isotropic. When the actual elements are non-isotropic, the total field can be

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formed by multiplying the array factor of the isotropic sources with the field of a single element.

The far-field radiation pattern of the array ($FF(\phi, I, r)$) is then given by [1]

$$FF(\phi, I, r) = EP(\phi, I, r).AF(\phi, I, r) \quad (1)$$

where $EP(\phi, I, r)$ gives the element pattern (in this work; the elemental pattern is considered to be $\cos(\phi)$ type) while $AF(\phi, I, r)$ is the array factor of the isotropic source.

With reference to Fig. 1, the array factor, $AF(\phi, I, r)$ for the CCAA in *x-y* plane may be written as [12-13]:

$$AF(\phi, I, r) = \sum_{m=1}^{M} \sum_{i=1}^{N_m} I_{mi} \exp\left[j\left(kr_m \sin\theta \cos(\phi - \phi_{mi}) + \alpha_{mi}\right)\right] \quad (2)$$

where

I_{mi} = current excitation of the i^{th} element of the m^{th} ring;

$k = 2\pi/\lambda$; λ being the signal wave-length;

ϕ_{mi} =angular separation between elements measured from positive x-axis.

If the elevation angle, $\theta = 90^0$ then (2) may be written as a periodic function of ϕ with a period of 2π radian. The angle is the element to element angular separation measured from the positive x-axis. The elements in each ring are assumed to be uniformly distributed. and are also obtained from [12] as:

$$\phi_{mi} = 2\pi\left((i-1)/N_m\right) \quad (3)$$

$$\alpha_{mi} = -Kr_m \cos(\phi_0 - \phi_{mi}) \quad (4)$$

ϕ_0 is the value of ϕ where peak of the main lobe is obtained. After defining the far-field radiation pattern, the next step in the design process is to formulate the objective function which is to be minimized.

The objective function "Cost-Function" (CF) may be written as :

$$CF = W_{F1} \times \frac{\left|FF(\phi_{msl1}, I_{mi}, r_m) + FF(\phi_{msl2}, I_{mi}, r_m)\right|}{\left|FF(\phi_0, I_{mi}, r_m)\right|} + W_{F2} \times \left(FNBW_{computed} - FNBW\ (I_{mi} = 1)\right) \quad (5)$$

FNBW is the First Null Beam Width. W_{F1} and W_{F2} are the weighing factors. Minimization of CF means maximum reductions of SLL both in lower and upper bands and lesser $FNBW_{computed}$ as compared to $FNBW\ (I_{mi} = 1)$. The evolutionary techniques described below are employed for the optimisation of the current excitation weights as well as the radii of the concentric rings. This results in the minimization of and hence reductions in both SLL and *FNBW*.

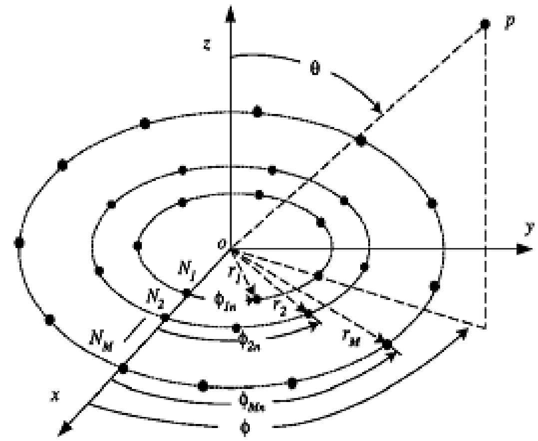

Figure 1. Concentric circular antenna array (CCAA)

III. Evolutionary Techniques Employed

A. Particle Swarm Optimization (PSO)

PSO is a flexible, robust population-based stochastic search/optimization technique with implicit parallelism, which can easily handle with non-differential objective functions, unlike traditional optimization methods. PSO is less susceptible to getting trapped on local optima unlike GA, Simulated Annealing, etc. Eberhart and Shi [14] developed PSO concept similar to the behavior of a swarm of birds. PSO is developed through simulation of bird flocking in multidimensional space. Bird flocking optimizes a certain objective function. Each particle (bird) knows its best value so far (pbest). This information corresponds to personal experiences of each particle. Moreover, each particle knows the best value so far in the group (gbest) among pbests. Namely, each particle tries to modify its position using the following information:

- The distance between the current position and the *pbest*
- The distance between the current position and the *gbest*

Similar to GA, in PSO techniques also, real-coded particle vectors of population n_p are assumed. Each particle vector consists of components or sub-strings as required number of current excitation weights and three radii, depending on the number of current excitation elements in each CCAA design.

Mathematically, velocities of the particle vectors are modified according to the following equation [12-13]:

$$V_i^{(k+1)} = w * V_i^{(k)} + C_1 * rand_1 * \left(pbest_i^{(k)} - S_i^{(k)}\right) + C_2 * rand_2 * \left(gbest^{(k)} - S_i^{(k)}\right) \quad (6)$$

where $V_i^{(k)}$ is the velocity of i^{th} particle at k^{th} iteration; *w* is the weighting function; C_1 and C_2 are the positive weighting factors; $rand_1$ and $rand_2$ are the random numbers between 0 and 1; $S_i^{(k)}$ is the current position of i^{th} particle vector at k^{th} iteration; $pbest_i^{(k)}$ is the personal best of i^{th} particle vector at k^{th} iteration; $gbest^{(k)}$ is the group best of the group at k^{th}

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iteration. The searching point in the solution space may be modified by the following equation:

$$S_i^{(k+1)} = S_i^{(k)} + V_i^{(k+1)} \quad (7)$$

The first term of (6) is the previous velocity of the particle vector. The second and third terms are used to change the velocity of the particle. Without the second and third terms, the particle will keep on "flying" in the same direction until it hits the boundary. Namely, it corresponds to a kind of inertia represented by the inertia constant, w and tries to explore new areas.

B. Particle Swarm Optimization with Constriction Factor and Inertia Weight Approach (PSOCFIWA)

Steps of PSOCFIWA as implemented for optimization of current excitation weights and location of elements are adopted from [15-16].

IV. Simulation Results and Discussions

This section gives the computational results for various CCAA designs obtained by PSO and PSOCFIWA techniques. For each optimization technique ten three-ring (M=3) CCAA designs are assumed. Each CCAA maintains a fixed optimal inter-element spacing between the elements in each ring. The limits of the radius of a particular ring of CCAA are decided by the product of number of elements in the ring and the inequality constraint for the inter-element spacing, $d, \left(d \in [\lambda/2, \lambda]\right)$. For all sets of experiments, the number of elements for the inner most ring is N_1, for outermost ring is N_3, whereas the middle ring consists of N_2 number of elements. For all the cases, $\phi_0 = 0^0$ is considered so that the centre of the main lobe in radiation patterns of CCAA starts from the origin. Since PSO techniques are sometimes quite sensitive to certain parameters, the parameters should be carefully chosen. Best chosen maximum population pool size, n_p=120, maximum iteration cycles for optimization, N_m=400. The PSO and PSOCFIWA generate a set of optimized non-uniform current excitation weights and optimal radii for each design set of CCAA. I_{mi} =1 corresponds to uniform current excitation. Sets of three-ring CCAA (N_1, N_2, N_3) designs considered for both without and with central element feeding are (2,4,6), (3,5,7), (4,6,8), (5,7,9), (6,8,10), (7,9,11), (8,10,12), (9,11,13), (10,12,13) and (11,13,15). Some of the optimal results are shown in Tables II-V. Table I depicts SLL values and FNBW values for all corresponding uniformly excited (=1) CCAA design sets.

Analysis of Radiation Patterns of CCAA

Figs. 2-3 depict the substantial reductions in SLL with optimal non-uniform current excitation weights and radii, as compared to the case of non-optimal uniform current excitation weights and radii (considering fixed inter-element spacing, d=λ/2) using PSO and PSOCFIWA optimization techniques respectively. All CCAA design sets having central element feeding (Case (b)) yield much more reductions in SLL as compared to the same not having central element feeding (Case (a)). It is depicted from Tables II-III, SLL reduces to -34.8 dB for case (a) and -42.8 dB for Case (b) using standard PSO algorithm for the CCAA having N_1=4, N_2=6, N_3=8 elements (Set No. III). Further for Tables IV-V, SLL reduces to -40.63 dB for Case (a) and *-46.4 dB (grand minimum SLL as determined by PSOCFIWA for Case (b)* and non-isotropic elements. This optimal design set along with central element feeding yields grand maximum SLL reduction among all the sets. The above reductions of SLL are determined by comparing Table I with Tables II-V. From comparison between Table II with Table IV and Table III with Table V it is clear that PSOCFIWA consistently yields better optimal results than standard PSO.

Table I. SLL And FNBW For Uniformly Excited (I_{mi} =1) CCAA Sets

Set No.	No. of elements in each rings (N_1,N_2,N_3)	Without central element (Case (a))		With central element (Case (b))	
		SLL (dB)	FNBW (deg)	SLL (dB)	FNBW (deg)
I	2, 4, 6	-14.82	128.52	-20.85	140.0
II	3, 5, 7	-15.44	107.32	-15.00	116.5
III	4, 6, 8	-13.97	90.14	-12.31	95.3
IV	5, 7, 9	-16.26	78.1	-15.99	81.54
V	6, 8, 10	-14.44	68.36	-16.12	71.22
VI	7, 9, 11	-13.15	60.92	-14.56	63.2
VII	8, 10, 12	-12.15	54.82	-13.37	56.34
VIII	9, 11, 13	-11.42	50.02	-12.49	51.18
IX	10, 12, 14	-10.86	46.02	-11.82	47.16
X	11, 13, 15	-10.42	42.22	-11.29	43.16

Table II. Current Excitation Weights, Radii, SLL and FNBW For Non-Uniformly Excited CCAA SETS (Case (a)) Using Standard PSO

Set No.	(I_{11}, I_{12}, ..., I_{mi}); (r_1, r_2, r_3) in λ	SLL (dB)	FNBW (deg)
III	0.7279 0.4510 0.6881 0.4139 0.3637 0.3369 0.1761 0.3218 0.3812 0.1436 0.2199 0.4047 0.2373 0.1150 0.1933 0.3614 0.1857 0.1432; 0.3278 0.5529 1.0164	-34.8	84.06
V	0.3926 0.8286 0.8617 0.8250 0.4137 0.6174 0.4013 0.0717 0.7292 0.5114 0.6028 0 0.4675 0.2144 0.6373 0.0000 0.1711 0.6595 0.0495 0.3955 0.0093 0.0000 0.4130 0.2356; 0.4521 0.8070 1.1075	-27.4	75.78
VII	0.7418 0.0158 0.7846 0.6196 0.5843 0.2871 0.5113 0.6138 0.7486 0 0.0001 0.7319 0.4079 0.4306 0.1283 0.1735 0.2601 0.3891 0.0153 0.4770 0.7203 0.5101 0.0036 0.3936 0.4705 0.1752 0.7871 0.0727 0.4921 0.4536; 0.7021 0.9603 1.4334	-31.4	53.28

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TABLE III. CURRENT EXCITATION WEIGHTS, RADII, SLL AND FNBW FOR NON-UNIFORMLY EXCITED CCAA SETS (CASE (B)) USING STANDARD PSO

Set No.	$(I_{11}, I_{12}, \ldots, I_{mi})$; (r_1, r_2, r_3) in λ	SLL (dB)	FNBW (deg)
III	0.9726 0.7591 0.8902 0.8270 0.8811 0.8091 0.8063 0.1988 0.8036 0.7994 0.2073 0.3948 0.1956 0.3990 0 0.3241 0.2292 0.3245 0; 0.3446 0.5475 0.8561	-42.8	99.38
V	0.1019 0 0.1320 0.0487 0.1133 0 0 0.1632 0 0.2594 0.4172 0.2690 0 0.1864 0.2906 0.0292 0.2487 0.2929 0 0.0128 0 0.2672 0.2255 0.0315 0.1578; 0.5051 0.7115 1.2015	-30.5	56.54
VII	0.4405 0.0179 0.3797 0.1236 0.6409 0 0.5578 0 0.5004 0.1658 0.1746 0.3087 0.2686 0.7212 0.2146 0.2409 0.1744 0.0215 0.4427 0.1972 0.0401 0.3352 0.0833 0.3630 0.5299 0.6256 0.0445 0.4637 0.0009 0.4620 0.3848; 0.7063 1.0696 1.3785	-32.1	59.49

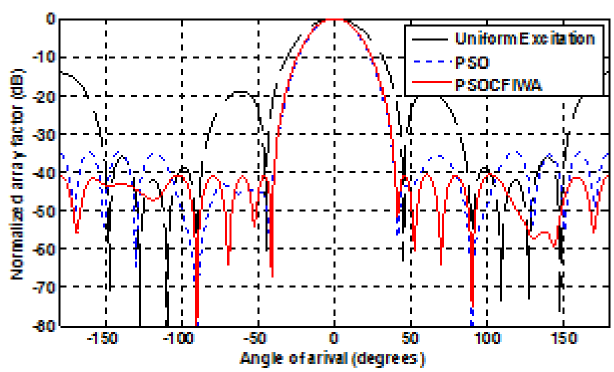

Figure 2. Radiation patterns for a uniformly excited CCAA and corresponding standard PSO and PSOCFIWA based non-uniformly excited CCAA Set No. III, Case (a)

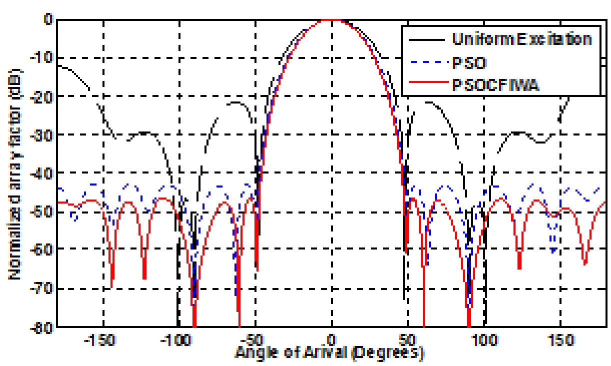

Figure 3. Radiation patterns for a uniformly excited CCAA and corresponding standard PSO and PSOCFIWA based non-uniformly excited CCAA Set No. III, Case (b)

TABLE IV. CURRENT EXCITATION WEIGHTS, RADII, SLL AND FNBW FOR NON-UNIFORMLY EXCITED CCAA SETS (CASE (A)) USING PSOCFIWA

Set No.	$(I_{11}, I_{12}, \ldots, I_{mi})$; (r_1, r_2, r_3) in λ	SLL (dB)	FNBW (deg)
III	0.4187 0.4324 0.4253 0.4641 0.5040 0.4870 0.7803 0.4756 0.4886 0.7873 0.3825 0.5105 0.3685 0 0.3432 0.4948 0.3638 0; 0.3080 0.5393 0.9312	-40.63	82.44
V	0.7824 0.6789 0.7448 0.7865 0.7369 0.8597 0.5666 0 0.4351 0.0503 0.3951 0 0.5887 0.2771 0.2367 0.4697 0.4185 0.1536 0.1200 0.1788 0.4241 0.5195 0.2748 0.1896; 0.5087 0.8147 1.2364	-30.99	62.46
VII	0.3745 0.0155 0.2730 0.6211 0.3072 0 0.5231 0.4277 0.3032 0.1700 0.0821 0.2573 0.5866 0.4136 0.0393 0.2053 0.3824 0.7337 0.3504 0.1253 0.4956 0 0.5273 0.4808 0.3770 0.1531 0.4814 0.2793 0.1950 0.3923; 0.7407 1.0453 1.3889	-33.17	59.04

TABLE V. CURRENT EXCITATION WEIGHTS, RADII, SLL AND FNBW FOR NON-UNIFORMLY EXCITED CCAA SETS (CASE (B)) USING PSOCFIWA

Set No.	$(I_{11}, I_{12}, \ldots, I_{mi})$; (r_1, r_2, r_3) in λ	SLL (dB)	FNBW (deg)
III	0.5036 0.3017 0.3472 0.3106 0.3461 0.5258 0.5174 0.3849 0.5187 0.5254 0.3787 0.2472 0.2292 0.2593 0 0.2573 0.2381 0.2520 0; 0.3313 0.5103 0.8743	-46.4	97.81
V	0 0 0 0.3198 0 0 0.0927 0.2215 0 0.4213 0.6739 0.3907 0 0.2300 0.6072 0.0790 0.2683 0.2745 0.1580 0 0.1401 0.2375 0.2903 0.0511 0.0182; 0.5311 0.7465 1.1001	-31.02	65.63
VII	0.1302 0.3603 0.3214 0.2918 0.5841 0.1007 0.3525 0.2632 0.5604 0.5855 0.0158 0.2466 0.5501 0.7566 0.6495 0.0710 0.0049 0.4405 0.7953 0.0682 0.2030 0.4934 0.2403 0.1782 0.2672 0.4127 0.2118 0.4630 0.1214 0.4503 0.0408; 0.6697 0.9655 1.3393	-32.43	60.00

Convergence profiles of PSO and PSOCFIWA

The minimum CF values are plotted against the number of iteration cycles to get the convergence profiles for the optimization techniques. Figs. 4-5 show the convergence profiles for PSO and PSOCFIWA for Set No. III, Case (b) CCAA respectively. PSO yields suboptimal higher values of CF but PSOCFIWA yields true optimal (least) CF values consistently in all cases. With a view to the above fact, it may finally be inferred that the performance of PSOCFIWA technique is better as compared to standard PSO. All optimization programs are written in MATLAB 7.5 version

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on core (TM) 2 duo processor, 3.00 GHz with 2 GB RAM.

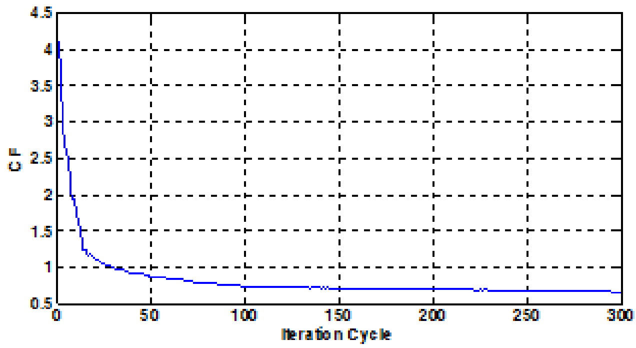

Figure 4. Convergence curve for standard PSO in case of non-uniformly excited CCAA (Set No. III, Case (b))

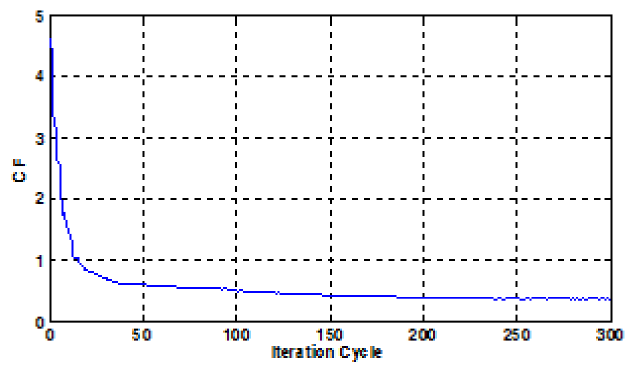

Figure 5. Convergence curve for PSOCFIWA in case of non-uniformly excited CCAA (Set No. III, Case (b))

V. Conclusion

This paper illustrates how to model the optimal design of non-uniformly excited CCAAs with optimal radii and non-isotropic elements for maximum SLL reduction using Standard PSO and PSOCFIWA. Computational results reveal that the optimal design of non-uniformly excited CCAA offers a considerable SLL reduction with respect to the corresponding uniformly excited and d=ë/2 inter-element spacing CCAA. The main contribution of the paper is threefold: (i) All CCAA designs having central element feeding yield much more reductions in SLL as compared to the same not having central element feeding, (ii) The PSOCFIWA technique outperforms significantly as compared to standard PSO in terms of solution optimality. iii) The CCAA design having N_1=4, N_2=6, N_3=8 elements along with central element feeding and non-isotropic elements gives the grand minimum SLL (-46.4 dB) as compared to all other design sets as determined by PSOCFIWA, which one is thus the grand optimal design set among all the three-ring designs.

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